Efficient Evaluation of the Field Functions of Soft Objects Using Interval Tree

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Abstract
We present an algorithm to evaluate the field function of a soft object efficiently. Instead of using a global field function that is defined by the sum of all local field functions, we consider only the set of local field functions that affects a point at which we want to evaluate the field function. To find the affecting local field functions efficiently, we exploit a data structure called interval tree based on the bounding volume of the component corresponding to the primitives (skeletons) of a soft object. The bounding volume of each component is generated with respect to the radius of a local field function of the component, threshold value, and the relations between the components and other neighboring components. The proposed scheme of field function evaluation can be used in many applications for soft objects such as modeling and rendering, especially in interactive modeling process.

1. Introduction
A soft object, also known as a blobby object or metaball, is a kind of implicit surface that is a widely used model to represent smooth geometric objects\(^1\). A soft object is defined by following parameters:

- **Primitives:** A primitive (also known as skeleton) \(p_i\) is usually a simple geometric object such as point and line segment. The set of the primitives determines the approximate shape of the soft object. In this paper, we mainly consider the convex primitives such as points, line segments, and triangles. The extension of our approach to any general type of geometric primitive is straightforward: for example, a concave polygon can be subdivided into a set of convex polygons, and a curve can be approximated by a piecewise linear curve.

- **Field function:** For each primitive \(p_i\), a local field function \(f_i\) is given, which maps a point in \(R^3\) into a value in \([0..1]\). Let \(p_i\) be a point primitive. The value of the local field function \(f_i\) at an arbitrary point \(v = (x, y, z)\) is computed by \(f_i(d_i)\), where \(d_i = \text{Euclidean distance between } v \text{ and } p_i\). Usually, \(f_i(d_i)\) is a smoothly decreasing function (see Figure 1). The sum of all local field functions included in a soft object, \(f = \sum f_i\), is called a global field function (simply field function) of the soft object.

- **Threshold:** A global constant \(T\) is given as a threshold value of a soft object. The surface of a soft object consisting of \(n\) primitives is defined by the set of points \(v\) satisfying
  \[ f(v) = \sum_{i=1}^{n} f_i(d_i) = T, \]
  where \(d_i\) is a distance between \(v\) and \(p_i\).

- **Radius:** The local field function in Figure 1(a) has a positive value for infinite domain \((d > 0)\), which can influences a point located very far from a primitive. To reduce the computational cost, a local field function like a Figure 1(b) is suggested, by which the influence of the local field function can be restricted to the domain, \(0 \leq d < r\).

We denote a three-tuple \(s_i = (p_i, f_i, r_i)\) a component of a soft object. A soft object, \(s\), is specified by a set of \(n\) components \(\{s_i = (p_i, f_i, r_i)\}, i = 1, \ldots, n\), and a threshold value \(T\).

Many algorithms such as polygonization and ray tracing of a soft object include a number of evaluations of the field...
function of the soft object. Thus, speeding up the evaluation of the field function of a soft object has been considered as an important problem. Let us assume that we want to evaluate the global field function \( f(v) \) at an arbitrary point \( v \). When we evaluate \( f(v) \) using Equation (1), we do not have to consider all local field functions \( f_i \), since \( f_i(d_i) \) vanishes, if \( d_i \) is greater than the radius \( r_i \). Thus, finding a set of primitives (called influencing primitives), whose distances from a given point \( v \) are less than the corresponding radii of the primitives, is a key problem of the fast evaluation of a field function.

In this paper, we suggest an efficient method to find the primitives that influence a given point using interval tree. The interval tree is a kind of binary search tree whose internal nodes store boundary values of the bounding volumes of the components and leaf nodes store a list of primitives that influence the corresponding interval. The bounding volume of a component is computed by considering the amount of the offset from the primitive in the component. The offset is determined to build compact bounding volumes in terms of the radius of the component and the relations between the neighboring components and the considering component. If the distance between a point and a primitive is less than the offset, then the point lies inside the bounding volume of the corresponding component of the primitive, while the inverse does not hold. Using the interval tree, the procedure of finding the influencing primitives for a point can be implemented by simple one dimensional searches in the interval tree. All distance computations are transferred to the preprocessing step that computes an interval tree, and thus, we do not have to compute the distance between a point and a skeleton in the process of the evaluation of a field function.

By using the interval tree, we can reduce the execution time of many applications such as polygonization and rendering of soft objects. In Section 6, we demonstrate the effect of our method by applying the method to the polygonization of soft objects. By the comparisons among our method, the simple evaluation method without any data structure such as interval tree, and other methods for speeding up the evaluation of the field function, the efficiency of our method is proved.

This paper is organized as follows. In Section 2, the researches on the field function of a soft object, the methods of the efficient evaluation of field function, and the bounding volumes in collision detection and ray tracing are reviewed. The algorithm for computing the bounding volume of a component is proposed in Section 3 and the algorithm for building and traversing an interval tree is illustrated in Section 4. In Section 6, implementation details and performance comparison with the conventional method for field function evaluation are provided. Finally, we conclude the paper and suggest some future works in Section 7.

2. Previous Work

In 1982, Blinn proposed a soft object, a new modeling paradigm using distribution function. The distribution function, known as field function or potential function, has played a major role in modeling and rendering models represented using soft objects. The first field function of a soft object, proposed by Blinn, is defined as follows:

\[
 f(d) = \frac{1}{2} \exp(-4\alpha d^2),
\]

where \( d \) is the distance between a vertex and a primitive and \( \alpha \) is a hardness factor that controls the slope of the function at \( d = 0.5 \). By this function, a vertex located very far from a primitive is still influenced by the primitive.

The inefficiency of the Blinn’s distribution function of infinite domain was improved by the field function having finite domain, proposed by Nishimura et al. Nishimura’s field function is defined by

\[
 f(d) = (d < 1) \times (d < \frac{1}{3}) \times \frac{4}{3} - 4d^2 : 2(1 - d)^2 : 0,
\]

where \( d \) is a normalized distance by the radius of the primitive. Using this function, the function evaluations for the vertices whose normalized distance is greater than 1 are avoided. Note that the function is \( C^2 \) continuous everywhere.

Nishimura’s function, however, possess inefficiency by computing square roots in its formula. Improved field functions in polygonal formula are proposed by Wyvill et al. and Murakama. These finite field functions, however, do not include the hardness factor that controls the slope of the function. Gascuel, Kacic-Alesic and Wyvill, and Blanc and Schlick proposed finite field functions with the hardness factor.

Wyvill et al. also presented some techniques with data structures for the fast evaluation of a field function. In their method, the volume of space of the whole scene is divided into cubes of fixed size. Each of these cubes is considered as an entry of a hash table, and the entry has a list of pointers to the primitives that influence the cube. Since the method decomposes a space, the method can be considered as one of

\[
\text{Figure 1: Field functions with (a) infinite domain, and (b) finite domain}
\]
image-space approaches. When a field function is evaluated at a given point \( p \), the cube containing \( p \) can be found easily from the coordinates of \( p \) (they explained this in terms of hashing), and thus, we can easily access to the primitives that influence \( p \).

Nevertheless, Wyvill et al.’s method has some disadvantages. First, it is not easy to determine the optimal size of a cube. Especially, when a number of components lie on a very small region, the components form a cluster in the hash table. Consequently, the method cannot effectively solve the problem of the fast evaluation of field function. If the size of a cube is too small, we need much amount of storages and the construction of the hash table structure takes long time. Second, the construction of the hash table structure is not easy especially for the primitives that are not points. For example, it is not easy to check exactly whether a line segment primitive influence a cube or not. The exact influence field of a line segment primitive may be the volume of a cylinder with arbitrary orientation. The intersection check between the cylinder and the cube may be cumbersome in the construction process. Thus, a simple approximation (such as axis-parallel hexahedron or sphere) of the exact influence field is often used. Third, Wyvill et al. did not consider the dynamic modification of the data structure. The insertion of a component is straightforward. However, if a component needs to be deleted from a soft object, the whole hash table must be scanned in the worst case.

Being compared with the Wyvill et al.’s method, our method using the interval tree is an object-space approach. Instead of decomposing the space, we build an interval tree based on the bounding volume of each component. This object-space based approach guarantees that the method can effectively handle the situation where many components form a cluster. In our method, the bounding volume can be defined according to more than 3 principle axes. Thus, the bounding volumes can be more exact approximations of the exact influencing fields. Furthermore, we seriously consider the dynamic update of tree interval tree to support the interactive modeling process. The insertion/deletion of a component into/from an interval tree can be easily implemented using simple algorithms for the binary search tree. The balancing based on AVL tree always maintain the interval tree to have an optimal height without any serious computation.

Researchers on collision detection and ray tracing have been traced a slightly different problem:

“Given two sets of objects, find intersecting objects without testing all pairs of the objects.”

Among the various methods proposed to solve the problem, a bounding volume hierarchy tree is a tree whose leaf node stores an individual object and its bounding volume. A parent node of the tree stores a bounding volume of the bounding volumes of its child nodes. The bounding volume hierarchy tree can be efficiently used especially in interactive applications to detect the collisions among the moving objects. The hierarchy of the tree can be used the level of collision detection with respect to the speed of the execution of the application.

3. Computing Bounding Volumes of an Object

3.1. Backgrounds

Various schemes for computing bounding volumes of geometric objects have been proposed in collision detection and ray tracing. The proposed bounding volumes are classified according to the complexity of the bounding volume geometry. The bounding volumes defined by some simple geometries, such as bounding sphere or axis-aligned bounding box, provide easy computation and fast comparison, while they are bad approximations by leaving large empty corners. On the contrary, bounding volumes using complex geometries, such as oriented bounding box or convex hull are tighter approximations than the simple ones, while they show poor performance in construction and checking intersections. To provide the benefits of the both types of the schemes, a hybrid approach, known as bounding slab, finite direction hull, or k-DOPs, have been proposed. The methods of this scheme computes tighter approximations than the simple ones and provide more efficient performance than the complex ones. In this paper, we build the bounding volumes of the components of a soft object based on this hybrid approach.

3.2. Definition of the bounding volume

For a component \( s_i = (p_i, f_i, r_i) \) of a soft object, \( k \) bounding directions are defined to compute the bounding volume of the component. In this paper, we use \( k = 3 \) or \( k = 7 \) for bounding directions. When \( k = 3 \), the three directions are \( (1, 0, 0), (0, 1, 0) \), and \( (0, 0, 1) \). When \( k = 7 \), the seven directions are \( (1, 0, 0), (0, 1, 0), (0, 0, 1), (1, 1, 1), (1, -1, 1), (-1, 1, 1), \) and \( (-1, -1, 1) \). In the case of \( k = 3 \), the resulting bounding volume coincides with the axis-aligned bounding box. For each bounding direction \( d_j \), the two planes \( P_j^+ \) and \( P_j^- \) that are orthogonal to the direction and circumscribed out the bounding volume are defined by

\[
P_j^+ = \{ x \mid \tilde{d}_j \cdot x = \tilde{v}_{\text{max}} \cdot d_j + \delta_j^+ \},
\]

\[
P_j^- = \{ x \mid \tilde{d}_j \cdot x = \tilde{v}_{\text{min}} \cdot d_j - \delta_j^- \},
\]

where \( \tilde{v}_{\text{max}} \) and \( \tilde{v}_{\text{min}} \) are vertices on \( p_i \) (the primitive of \( s_i \)) whose inner product with \( \tilde{d}_j \) is maximum and minimum, respectively. Figure 2 shows an example of the computation of \( P_j^+ \) and \( P_j^- \) for various primitives with the definitions of \( \tilde{v}_{\text{min}} \) and \( \tilde{v}_{\text{max}} \).

\( \delta_j^+ \) and \( \delta_j^- \) are offsets that indicate the range of influence...
of \( s_i \) to the direction of \( \delta_{j+} \) and \( \delta_{j-} \), respectively (see next subsection for the definition of offset). Consequently, the intersection of two halfspaces that enclose the object, denoted as \( H_j \), is defined by

\[
H_j = \{ x | v_{min} \cdot d_j - \delta_{j-} \leq d_j \cdot x \leq v_{max} \cdot d_j + \delta_{j+} \}.
\]

Two values \( b_{j-} = v_{max} \cdot d_j - \delta_{j-} \) and \( b_{j+} = v_{max} \cdot d_j + \delta_{j+} \) are called the bounding values of \( H_j \). Therefore, a bounding volume of the primitive \( p_i \), denoted as \( H_i \), using \( k \) bounding directions are defined as follows:

\[
H_i = \bigcap_{j=1}^{k} H_j.
\]

Suppose we test the intersection between the bounding volumes of two components, \( s_n \) and \( s_m \). The corresponding bounding volumes, \( H^n \) and \( H^m \), are intersected, if \( h^n_j \) and \( h^m_j \) are intersected, for all \( j = 1, 2, \ldots, k \). Note that the intersection between \( h^n_j \) and \( h^m_j \) can be tested by comparing the boundary values.

### 3.3. Calculation of the offset

\( \delta_{j+} \) and \( \delta_{j-} \) are offsets of a component \( s_i \) in direction of \( d_j \) and \( -d_j \), respectively. Basically, since we define the bounding volume to find influencing primitives, the offset is to be defined as \( r_i \), the radius of \( s_i \). In many cases, however, the actual range of influence of \( s_i \) becomes \( r_i' = f_{i}^{-1}(T) \), which is the actual radius of \( s_i \). Note that \( f_{i}(r_i') = T \) is a boundary of influence. Figure 3 compares \( r_i' \)'s for various \( T \). Note that \( r_i' \) coincides with \( r_i \) at \( T = 0 \).

To build a compact bounding volume of an object, we compute the offset according to the actual range of influence of the object. Since the actual range of influence varies according to the threshold value and the relations between neighboring objects, we define the relation between two soft objects before defining the offset. Let \( \rho_{nm} \) denote a vector emanating from a point on a primitive \( p_n \) to a point on \( p_m \), where \( \| \rho_{nm} \| \), the magnitude of \( \rho_{nm} \), is the shortest distance between \( p_n \) and \( p_m \). Since we assume that the primitives are convex, at least one of the two end points of \( \rho_{nm} \) is a vertex of a primitive (see Figure 4). The relation between two components \( s_n \) and \( s_m \) can be classified according to \( \rho_{nm} \), \( r_n \) and \( r_m \) as follows:

1. \( s_n \) is related to \( s_m \), if \( \| \rho_{nm} \| \leq r_n + r_m \).
2. \( s_n \) and \( s_m \) are disjoint, otherwise.

Figure 5 shows the classification of types according to the radii of the components.

Initially, the offsets in all bounding directions of a component \( s_n \) are set to the actual radius \( r_n' \) of the object. Then, all components are tested pairwisely to check whether they are related or not. For two related objects \( s_n \) and \( s_m \), \( \delta_{j+}^{nm} \), the offset of \( s_n \) in the bounding direction of \( d_j \), is set to \( r_n \), if \( s_m \) is in the direction of \( d_j \) by testing \( d_j \cdot \rho_{nm} \geq 0 \). Otherwise, \( \delta_{j+}^{nm} \) is set to \( r_m \). The algorithm that computes offsets for all objects and bounding directions are illustrated in Figure 6. In Figure 5, various shapes of bounding volumes determined from the offsets are illustrated.

### 4. Interval Tree

#### 4.1. Definition of interval tree

Let \( x \) be a soft object consisting of \( N \) components \( s_i \), \( i = 1, \ldots, N \). The bounding volumes of each component \( s_i \) is

\[
H_i = \bigcap_{j=1}^{k} h^i_j,
\]

where \( k \) is the number of bounding directions. As we have referred to in Section 3.2, the two bounding values of \( h^i_j \) are denoted by \( b^i_j+ \) and \( b^i_j- \), respectively.

For the soft object \( x \), we construct \( k \) interval trees each of which corresponds to a single bounding direction. Let us...
consider an interval tree \( IT_j \) of a specific direction \( j \) such that \( 1 \leq j \leq k \). There are \( 2N \) boundary values \( b_{j,\text{\neg}}^i \) and \( b_{j,\text{\neg}}^i \) of \( h_j^i \), for all \( i = 1, \ldots, N \). For simplicity, we assume that all \( b_{j,\text{\neg}}^i \) and \( b_{j,\text{\neg}}^i \), for \( i = 1, \ldots, N \), are distinguished, and \( v_q \)'s, \( q = 1, \ldots, 2N \), denote a sorted list in ascending order of the boundary values. Then, \( IT_j \) is defined as follows:

1. The subset of \( IT_j \) including all internal (non-leaf) nodes and connecting edges between them is a one dimensional binary search tree. Thus, each internal node stores a bounding value \( v_q \), \( 1 \leq q \leq 2N \).

2. Let \( I \) be a leaf node, and the parent of \( I \) be denoted by \( p(I) \) storing a value \( v_q \). The leaf node \( I \) has a set of influencing components denoted by \( c(I) \) defined as follows:

a. When \( I \) is a left child of \( p(I) \),

i. If \( q = 1 \), \( c(I) = \emptyset \).

ii. Otherwise, \( c(I) \) is a set of the components influencing the region \( \{ v_{q-1}, v_q \} \).

b. When \( I \) is a right child of \( p(I) \),

i. If \( q = 2N \), \( c(I) = \emptyset \).

ii. Otherwise, \( c(I) \) is a set of the components influencing the region \( \{ v_q, v_{q+1} \} \).

The data structure of an internal node is described in Figure 7. The value field stores the bounding value of the node.

Figure 5: Relations of two components \( s_n \) and \( s_m \) according to their default and actual radii, and the shapes of bounding volumes \( H_n \) and \( H_m \)

Compute offset (Soft objects \( s_1, s_2, \ldots, s_N \), bounding directions \( d_1, \ldots, d_k \))

\[
\begin{aligned}
&\text{for ( } n = 1 \text{ to } N \text{ )} \\
&\quad \text{for ( } j = 1 \text{ to } k \text{ )} \\
&\quad \quad \delta_{j,\text{\neg}}^n \Leftarrow d_j^n; \\
&\quad \quad \delta_{j,\text{\neg}}^n \Leftarrow r^n; \\
&\quad } \\
&\text{for ( } n = 1 \text{ to } N \text{ )} \\
&\quad \text{for ( } m = 1 \text{ to } N \text{ )} \\
&\quad \quad \text{if ( } n \neq m \text{ )} \\
&\quad \quad \quad \varrho_{nm} \Leftarrow \text{the shortest distance vector from } p_n \text{ to } p_m; \\
&\quad \quad \quad \text{if ( } \| \varrho_{nm} \| \leq r_n + r_m \text{ )} \\
&\quad \quad \quad \quad \text{for ( } j = 1 \text{ to } k \text{ )} \\
&\quad \quad \quad \quad \quad \text{if ( } \varrho_{nm} \cdot \delta_{j,\text{\neg}}^n > 0 \text{ )} \\
&\quad \quad \quad \quad \quad \quad \delta_{j,\text{\neg}}^n \Leftarrow r_n; \\
&\quad \quad \quad \quad \quad \text{else \ } \delta_{j,\text{\neg}}^n \Leftarrow r_n; \\
&\quad \quad } \\
&\end{aligned}
\]

Figure 6: Algorithm for computing offsets

The reference_count field, which is initialized with 1, of a node stores the number of times of appearing the boundary value \( v_q \) in the list of \( v_q \)'s, \( q = 1, \ldots, 2N \). Thus, we do not have \( 2N \) internal nodes when any two or more boundary values are the same. If a left (resp. right) child of an internal node is a leaf node, larray (resp. rarray) pointer points to a leaf node, i.e., an array of component pointers. Thus, in this data structure, a leaf node \( l \) represents a set \( c(l) \) directly. In Figure 4.1 an example interval tree from 7 objects is illustrated. In the figure, a component is indexed as \( s \) instead of \( s_i \) for the convenience of notation. Also, for simplicity, boundary values are denoted by \( b_m \) and \( b_{M} \) instead of \( b_{j,\text{\neg}}^i \) and \( b_{j,\text{\neg}}^i \), respectively.

4.2. Building an Interval Tree

The algorithm for building an interval tree is composed of three phases: i) sorting all boundary values, ii) building a binary search tree from the sorted list of the boundary values, and iii) inserting each component to the leaf nodes that correspond to the intervals where the component influences. To build a balanced interval tree, the median value of the boundary values are selected and inserted to the interval tree recursively. When a value is inserted, if the value field of
struct Node
{
    float value;
    int reference_count;
    Node *left, *right;
    Object **larray, **rarray;
}

**Figure 7:** Definition of an internal node

the visited node is identical to the value, then the inserting process stops by increasing the reference_count field of the node by 1. An interval tree from $N$ components with $2N$ distinguished boundary values has $2N$ internal nodes and $2N + 1$ leaf nodes. Note that the number of leaf nodes, which is the number of arrays, coincide with the number of intervals that divide space with $2N$ values. The correspondence between the interval and the leaf node must be maintained for all modifications of the interval tree.

The rule of inserting a component $s_i$ having two boundary values $b_{i-}$ and $b_{i+}$ for a bounding direction $j$ to an interval tree $IT_j$ (the third phase of the building algorithm) is as follows. At an internal node having a value $v_q$, $b_{i-}$ and $b_{i+}$ are compared with $v_q$, and one of the following actions is taken.

- If $b_{i-} < b_{i+} \leq v_q$, then $s_i$ is propagated down to the left child of the node.
- If $b_{i+} > b_{i-} \geq v_q$, then $s_i$ is propagated down to the right child of the node.
- If $b_{i-} < v_q < b_{i+}$, $s_i$ is propagated down to the both of left child and right child.

After reaching a leaf node, the pointer to $s_i$ is inserted to the array in the leaf node. An algorithm of inserting a component to an interval tree is illustrated in Figure 8.

**4.3. Field function evaluation using interval trees**

When a field function is evaluated at a point $p$, $p$ is projected onto each bounding direction $d_j$. Then, the one dimensional projected coordinates, $d_j \cdot p$, traverse down the corresponding interval tree $IT_j$ to find an interval where the coordinates belong. An algorithm of searching the set of influenc-
Thus, the interval tree takes $2N$ components to the interval tree takes $O(n)$.

The size of the array in each leaf node can be output-dependent. That is, if a component is inserted to an interval tree, the running time of the range searching problem is $O(n)$.

The Eurographics Association 2000.

The evaluation of a field function, the interval tree that is completely balanced is traversed from the root to a leaf node, which takes $O(\log N)$ time.

5. Dynamic Update of Interval Tree

In interactive modeling process of a soft object, the components are inserted, deleted or edited dynamically. In such cases, the interval tree should be updated appropriately. We will consider the following topics for dynamic update of an interval tree:

- Insertion of a new component into an interval tree,
- Deletion of an existing component from an interval tree,
- Balancing an interval tree.

Note that the deletion of a component can be implemented by the combination of a deletion and an insertion.

5.1. Inserting a new component

When a new component is inserted into an interval tree, the two boundary values of the component are inserted into the tree as two internal nodes. We can insert a bounding value using the conventional binary search algorithm utilized in the previous section. Let $i$ be a new internal node that stores the inserted bounding value is created. Assume that $i$ will be inserted as the left child of an existing node $p$ that currently has a left child $l(p)$, where $l(p)$ is a leaf node. Then, the following steps are applied to insert $i$:

1. Disconnect $l(p)$ from $p$.
2. Make $i$ be the left child of $p$.
3. Duplicate $l(p)$. Let $l'(p)$ be a copy of $l(p)$.
4. Make $l(p)$ and $l'(p)$ be the left and right child of $i$, respectively.

For the other case, when $i$ will be the right child of $p$, the similar steps can be used. After two new internal nodes are created, the component itself is inserted to the components arrays in the leaf nodes of the interval tree using the algorithm in Figure 8. Figure 10 shows an interval tree after the insertion of new component $s^7$. The original interval tree is shown in Figure 4.1.

5.2. Deleting a component

When a component is deleted from an interval tree, first, the component pointers stored in the arrays of the leaf nodes are deleted. To delete the component pointers in the arrays, we apply an algorithm that is similar to the insertion algorithm in Figure 8. The only difference is that on reaching an array, the component pointer is deleted instead of inserted. Next, we traverse the tree to delete the internal nodes corresponding to two bounding values of the component. When we reach an internal node $i$ of a bounding value, if $i$ has reference_count > 1, then the traversal stops by decreasing the reference_count by 1. Otherwise, $i$ is deleted and the tree is updated as follows.

![Figure 9: Algorithm for searching an object into an interval tree](image)

<table>
<thead>
<tr>
<th>Algorithm Search_IntervalTree</th>
</tr>
</thead>
<tbody>
<tr>
<td>( float $v_j$, Node *$nd$ )</td>
</tr>
<tr>
<td>{</td>
</tr>
<tr>
<td>if ( $v_j \leq nd$.value ) {</td>
</tr>
<tr>
<td>if ( $nd$.array $\neq$ NULL )</td>
</tr>
<tr>
<td>return $nd$.array;</td>
</tr>
<tr>
<td>else</td>
</tr>
<tr>
<td>Search_IntervalTree ( $v_j$, $nd$.left );</td>
</tr>
<tr>
<td>}</td>
</tr>
<tr>
<td>else {</td>
</tr>
<tr>
<td>if ( $nd$.array $\neq$ NULL )</td>
</tr>
<tr>
<td>return $nd$.array;</td>
</tr>
<tr>
<td>else</td>
</tr>
<tr>
<td>Search_IntervalTree ( $v_j$, $nd$.right );</td>
</tr>
<tr>
<td>}</td>
</tr>
</tbody>
</table>

Algorithm Search_List ( Vertex $\tilde{v}$ )

List $l_1$, $l_2$, ..., $l_k$;
IntervalTree $IT_1$, $IT_2$, ..., $IT_k$;

for ( $j = 1$ to $k$ ) { |
  $l_j$ $\leftarrow$ Search_IntervalTree ( $\tilde{v}$, $\tilde{d}_j$, $IT_j$ ); |
} return Intersection ( $l_1$, $l_2$, ..., $l_k$ );

(c) The Eurographics Association 2000.
1. If both of two children \( l(i) \) and \( r(i) \) of \( i \) are leaf nodes,
   a. Merge the two arrays of component pointers in \( l(i) \) and \( r(i) \) and make a single leaf node \( nl \) storing the merged array.
   b. Substitute \( nl \) for \( i \).
   c. Purge \( i \).
2. Otherwise,
   a. If \( l(i) \) is not a leaf node,
      i. Find an internal node \( j \) having the largest value in the left subtree of \( i \).
      ii. Copy the value and the reference count of \( j \) into \( i \).
      iii. If one of \( l(j) \) and \( r(j) \) is an internal node \( k \), substitute \( k \) for \( j \).
      iv. If both \( l(j) \) and \( r(j) \) are leaf nodes, make a merged leaf node \( nl \) as in Step 1(a), and substitute \( nl \) for \( j \).
      v. Purge \( j \).
   b. Otherwise, take a similar step to Step 2(a) with \( j \) = the internal node of the smallest value in the right subtree of \( i \).

Note that in Step 2-(a)-iii, both of \( l(j) \) and \( r(j) \) cannot be internal nodes at the same time, since \( j \) must be the largest value in the left subtree of \( i \). In Step 2-(a)-i, we only copy the value and the reference count of \( j \) to \( i \). An elegant thing is that we do not have to modify the original child of the node \( i \), even when the child is a leaf node. This is because we already pick out the component pointers to the deleted component before the deletion of internal nodes. That is, when both of the right child of \( i \) and \( j \) are leaf nodes, the arrays of the component pointers in the two leaf nodes are already the same after picking out the component pointers.

The above description shows that the correspondence between arrays and intervals are maintained after deleting a component from the tree. In Figure 11, we illustrate how an object, for example \( s^5 \), is deleted from the tree. Note that after deleting \( s^5 \), the interval tree becomes unbalanced.

### 5.3. Maintaining balance of the interval tree

Even though we built an initial balanced interval tree by inserting the median of the bounding values recursively, the dynamic modification may break the balance of the tree. We apply single rotation and double rotation, which are the techniques used in the AVL tree\(^\text{14}\). In Figure 11, since the unbalance is propagated in the alternating direction (\( b_{nl} \) is the right...
child of $b^k_m$ and $b^6_m$ is the left child of $b^4_M$), a double rotation is applied. After inserting an object $s^5$ to the resulting tree, another type of unbalance appears (See Figure 12). In this case, since the unbalance is propagated in the identical direction ($b^k_m$ is the left child of $b^k_m$ and $b^6_m$ is the left child of $b^4_M$), single rotation is applied. Through these methods, the balance of the interval tree is maintained.

5.4. Analysis of update algorithms

As well known, the balancing of AVL tree takes constant time \(^14\). By the balancing, we can always maintain the height of an interval tree as \(O(\log N)\), where \(N\) is the number of components. The running time of the insertion algorithm depends on the time of the algorithm described in Figure 8 for inserting a component pointers into the appropriate leaf nodes. As we referred to in Section 4.4, the algorithm takes \(O(\log N + N) = O(N)\) times in the worst case. The running time of the deletion algorithm is \(O(\log N)\), which is the same as that of the conventional deletion algorithm in binary search tree \(^14\).

6. Implementation and Results

The proposed algorithm in this paper is implemented at a PC environment with Pentium-III CPU 500 MHz and 256 MB memories. The software environment is Visual C++ with MFC and OpenGL libraries. To measure the performance of the proposed algorithm to that of the conventional ones, we apply the algorithm for polygonizing soft objects. Marching cube-based conventional algorithm \(^11\) is implemented for the polygonization. The marching cube method polygonizes the soft objects by decomposing space into small-sized cells and generating intersecting polygons with the cells and the objects. We designed two examples, each of which is composed of point skeletons and line segment skeletons, respectively. The first example is a smooth object composed of two loops. Each loop is approximated by line segments, which are applied as the primitives of soft objects. From this example, we can conclude that even though the number of generated polygons are nearly same, the number of primitives is an affecting factor for the performance. The measured performance and the statistics for the influencing primitives are illustrated in \(^13\). Notice that the speed up of the proposed algorithm is due to the reduced number of influencing primitives. The second example is a mesh of point skeletons. In this example, we fix the number of primitives and change the number of generated polygons by controlling resolution of the cells. It is natural that the increase of the number of cells indicates the increase of the computation time. In this case, the computation time increases much faster for the conventional algorithms. Some of the blank column is due to the low performance of the platform, which prohibited us from achieving results. The shape of the soft object with 1210 point skeletons are suggested in accompanying file 1210.jpg. With the picture, we present two movie files to show the result of this paper. The first file, named example1.mpeg, compares the performance of polygonization using interval tree with the hash table method proposed by Wyvill \(^15\). The second file, named example2.mpeg, shows an interactive modeling environment of soft objects, which are polygonized with the proposed method.

7. Conclusion and Future Work

In this paper, we proposed a new algorithm for evaluating field functions of soft objects. To remove the unnecessary distance computations of conventional field functions, we built the bounding volumes of the objects and decomposed space into intervals by designing interval tree. Using this tree, a set of influencing primitives for a vertex is found without computing the distances between the vertex and all primitives. We applied this new field function in polygonizing soft objects. The conventional marching cube-based polygonization method using the proposed field function shows faster and more consistent response time than that using the conventional field function.

We are investigating the problem of the localized polygo-
Min, Lee and Park / Interval Tree

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Figure 13: Comparison of performance with example presented in 1210.jpg

nization using interval tree, which is the application of our work suggested in this paper. When a soft object is updated, the polygonization of the soft object must be recomputed. Since the area where the field function changes can be classified using interval tree, we can repolygonize the soft object only in that classified area without repolygonizing the whole object. With this extension, the interval tree can be one of the solutions to the interactive modeling environment using soft objects.

References


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